

Polarizing Stored Beams by Interaction with Polarized Electrons

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Abstract

A polarized, internal electron target gradually polarizes a proton beam in a storage ring. Here, we derive the spin-transfer cross section for $\vec{e}(p, \vec{p})e$ scattering. A recent measurement of the polarizing effect of a polarized atomic hydrogen target is explained when the effect of the atomic electrons is included. We also consider the interaction of a stored beam with a pure electron target which can be realized either by a comoving electron beam or by trapping of electrons in a potential well. In the future, this could provide a practical way to polarize antiprotons.

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Storage rings are important tools in particle and nuclear physics research [1]. They are used for beam processing and subsequent slow extraction, as well as for experiments with thin, internal targets. Ring designs, almost always, incorporate a means of phase space cooling such as stochastic cooling [2] or electron cooling [3]. The study of spin dependence in nuclear reactions requires stored, polarized beams, and their behavior in rings has recently been studied in detail (see Ref. [4]).

Beam storage times in a ring can be hours or even days. During that time, stored particles orbit with frequencies of the order of several MHz, and small, additive spin-dependent interactions may affect the polarization of a stored beam. In one of the methods, which has been proposed, inhomogeneous magnetic fields gradually split the stored beam into components of given magnetic substates [5]. Another method utilizes the spin-dependent interaction of the stored ions with an aligned internal target. This scheme, also known as “filter method”, has recently been tested at the TSR ring at the Max-Planck Institute in Heidelberg [6]. In this test, an initially unpolarized 23 MeV proton beam interacting with an internal polarized hydrogen target was found to acquire polarization P_B at the rate of $dP_B/dt = 0.0124 \pm 0.0006$ per hour. An attempt to explain the measurement was made by considering selective beam loss due to the spin-dependent part of the strong, pp total cross section, which, at this energy is $\sigma(\uparrow\uparrow) - \sigma(\downarrow\uparrow) = -122$ mb. The minus sign signifies that stored ions in the spin-down substate are lost more rapidly. The spin-up polarization of the surviving beam, calculated this way, is about twice as large as the measurement [6].

Let us now turn to a discussion of the interaction of a stored proton beam with a polarized, internal electron target. First, we derive the cross section for polarization transfer in $\vec{e}(p, \vec{p})e$ scattering. The calculation is carried out in the center-of-mass frame. The initial electron polarization is along \hat{i} , the scattered proton is polarized along \hat{j} , where \hat{i} , \hat{j} are unit vectors expressed in terms of the basis vectors \hat{l} (longitudinal), \hat{m} (sideways) and \hat{n} (normal to the scattering plane), as defined in Eq. (2.2) of Ref. [7]. We write m_p , \mathbf{p} and \mathbf{k} for the mass, the initial and the final momentum of the proton, and, respectively, m_e , \mathbf{p}_e and \mathbf{k}_e for the electron.

Starting from the expression for the center-of-mass cross section as given in Eq. (B.1) of Ref. [8], we write

$$\frac{d\sigma}{d\Omega} K_{j00i} \approx \frac{m_e^2}{4\pi^2} |M_{ij}|^2, \quad (1)$$

where K_{j00i} is the spin transfer coefficient from the target to the projectile (formally defined in Ref. [7]), and M_{ij} is the invariant matrix between the appropriate spin states in the initial and final state. First we evaluate the matrix element in plane-wave Born approximation (PWBA), and later we will discuss the inclusion of Coulomb distortions. The PWBA matrix element M is given by

$$M = \frac{4\pi\alpha}{q_\mu^2} i \bar{U}_k \Gamma_\mu U_p \bar{U}_{k_e} \gamma^\mu U_{p_e}, \quad (2)$$

where $q_\mu = k_\mu - p_\mu$ is the momentum transfer to the proton, and $\Gamma_\mu = \gamma_\mu + \lambda_p \frac{i\sigma_{\mu\nu}}{2m_p} q^\nu$ with $\lambda_p = 1.793$, the anomalous moment of the proton. The square of Eq. (2), summed over the spin of the final electron and averaged over the spin of the initial proton can be calculated in standard fashion [8]. The result is,

$$\frac{d\sigma}{d\Omega} K_{j00i} = -\alpha^2 \frac{(1 + \lambda_p)m_e}{q_\mu^2 m_p} \left\langle S_j \cdot S_i^e - \frac{q \cdot S_j q \cdot S_i^e}{q_\mu^2} \right\rangle. \quad (3)$$

Here, S_j and S_i^e are unit four-vectors describing the spins of the final proton and the initial electron along \hat{j} and \hat{i} . The average $\langle \Phi(S_i^e, S_j) \rangle$ in Eq. (3) is calculated according to the prescription $\langle \Phi(S_i^e, S_j) \rangle = \frac{1}{2}[\Phi_{++} + \Phi_{--} - \Phi_{+-} - \Phi_{-+}]$, where the first sign in $\Phi_{\pm\pm}$ indicates whether S_i^e points in the direction of, or opposite to, \hat{i} . The second sign is analogous for S_j and \hat{j} . For $\hat{i} = \hat{j} = \hat{n}$ as well as for $\hat{i} = \hat{j} = \hat{l}$, one finds $\langle \Phi(S_i^e, S_j) \rangle = -2$ [8]. For $\hat{i} = \hat{m}$ or $\hat{j} = \hat{m}$ $\langle \Phi(S_i^e, S_j) \rangle = 0$. This gives,

$$\frac{d\sigma}{d\Omega} K_{n00n} = -\frac{\alpha^2(1 + \lambda_p)m_e}{2p_e^2 m_p \sin^2(\theta/2)}, \quad (4)$$

also $K_{l00l} = K_{n00n}$, and $K_{m00m} = 0$. Here p_e is the momentum of the electron (in the center of mass frame). One also has $K_{j00i} = 0$ for $i \neq j$ and,

$$A_{00ji} = K_{j00i}, \quad (5)$$

where A_{00ji} is the correlation coefficient between the initial electron and proton spins, see Ref. [7].

The maximum scattering angle of a proton from an electron at rest is $m_e/m_p = 0.54$ mrad. This is well within the acceptance angle of any storage ring, thus protons scattering from electrons stay in the ring. Therefore, we are interested in the total spin cross section,

$$\sigma_{ij} \equiv \int_{\theta > \theta_{min}} \frac{d\sigma}{d\Omega} K_{j00i} d\Omega. \quad (6)$$

When Eq.(4) is used in Eq.(6), the total cross section diverges logarithmically for $\theta \rightarrow 0$. However, Eq. (4) must be modified at very small angles because of the screening of the Coulomb field caused by either the proton in the same H atom as the electron, or by the other electrons in a dense, pure electron target. This introduces in Eq. (6) a cutoff at a minimum scattering angle θ_{min} . In the above calculation we have evaluated a *quantum* effect which is proportional to the interference of the Coulomb and hyperfine amplitudes. The *classical* effect (proportional to the square of the hyperfine interaction) is very small.

For an atomic hydrogen target, beam protons interact with both electrons and target protons. For momentum transfers much greater than one over the Bohr radius a_0 we expect incoherent scattering from “almost free” constituents. If the electron and proton are both polarized in the same direction (spin-one atom) then,

$$\frac{d\sigma}{d\Omega} K_{j00i}|_{atom} \approx \frac{d\sigma}{d\Omega} K_{j00i}|_{pp} + \frac{d\sigma}{d\Omega} K_{j00i}|_{ep}, \quad (7)$$

for momentum transfers $q \gg 1/a_0$ while for smaller momentum transfers $q < 1/a_0$ the projectile sees a neutral atom and the cross section is suppressed, $\frac{d\sigma}{d\Omega} K_{j00i}|_{atom} \approx 0$. The *pp* transfer cross section (first term in Eq.(7)) has important contributions from Coulomb-nuclear interference. This will be discussed in another publication [9]. Here, we focus on the *ep* cross section (and drop the *ep* subscript in the rest of this paper).

At large impact parameter, the Coulomb field of the target electron is screened and there is no interaction. This can be taken into account by cutting off the integration in Eq.(6) at a minimum angle, θ_{min} . We define θ_{min} so that the minimum momentum transfer is the inverse of the screening distance,

$$2p_e \sin(\theta_{min}/2) = 1/\Lambda. \quad (8)$$

For an atomic target we take the screening distance to be the Bohr radius,

$$\Lambda_A = a_0 = 52900 \text{ fm.} \quad (\text{atom}) \quad (9)$$

Alternatively, for a pure electron target the screening length depends on the electron density n_e and can be estimated as the Debye length, $\Lambda \approx \Lambda_D = (kT/4\pi\alpha n_e)^{1/2}$ where T is the temperature of the electron gas. As an example we take $kT \approx 0.1 \text{ eV}$ and $n_e \approx 10^{10} \text{ cm}^{-3}$, so that,

$$\Lambda_D \approx 10^{10} \text{ fm.} \quad (\text{plasma}) \quad (10)$$

Note that this is a simple estimate: the screening may depend on the electron velocity distribution, the relative velocity between the protons and the electrons, and possible magnetic fields.

Using Eqs. (4),(6) and (8), the total cross section can now be evaluated,

$$\sigma_{nn} = - \left[\frac{4\pi\alpha^2(1 + \lambda_p)m_e}{p_e^2 m_p} \right] \ln(2p_e\Lambda). \quad (11)$$

This expression is valid even at high energies where the electrons (in the center of mass frame) are relativistic.

At low energies, corrections from Coulomb distortions become important. The use of distorted-waves in the Born approximation (DWBA) modifies the plane-wave result in two ways. First, the matrix element of the short-ranged hyperfine interaction is enhanced by (approximately) the square of the Coulomb wave function at the origin, $C_0^2 = 2\pi\eta/(\exp(2\pi\eta) - 1)$, where the Coulomb parameter $\eta = -z\alpha/v$ depends on the beam charge z and the relative velocity v . Second, there is an angle-dependent relative phase between the Coulomb and the hyperfine amplitudes. Together, these effects are responsible for an additional factor D on the right-hand side of Eq.(4),

$$D \approx C_0^2 \cos[\eta \ln(\sin^2(\theta/2))]. \quad (12)$$

Including this factor in the integration in Eq.(6) yields,

$$\sigma_{nn} = - \left[\frac{4\pi\alpha^2(1 + \lambda_p)m_e}{p_e^2 m_p} \right] C_0^2 \left(\frac{v}{2\alpha} \right) \sin \left[\frac{2\alpha}{v} \ln(2p_e\Lambda) \right]. \quad (13)$$

For antiproton-electron scattering, C_0^2 is simply evaluated for a *positive* η parameter. The spin transfer cross section σ_{nn} Eq. (13) for scattering of protons from transversely polarized electrons versus the laboratory kinetic energy is shown in Fig. 1. Note, that other transfer cross sections are simply related to σ_{nn} by $\sigma_{ll} = \sigma_{nn}$ and $\sigma_{mm} = 0$ (see below Eq. (4)). The

magnitude of the plane wave result (dotted line) for a pure electron target is larger than for an atomic target because for the latter the screening length is smaller and the integral over angle cuts off at a larger θ_{min} . Taking into account distortions leads to the solid line in the case of stored protons, and the dashed line for antiprotons. For an atomic target, distortions are mainly described by C_0^2 and lead to an enhancement (with respect to the PWBA result) of σ_{nn} for p, and a suppression for \bar{p} . The situation is different for a pure electron target for which (in our simple approximation) the screening length is much larger and the Coulomb phase is significant at the lower end of the angle integration. This reduces σ_{nn} , and, below 2.5 MeV causes a sign change. Indeed, in the limit of very small relative velocities the Coulomb phase averages to zero, insuring the correct (small) classical limit. We note that still, the transfer cross section can be as large as a barn.

As a beam circulates through an internal polarized electron target the polarization P_B of the beam changes according to [9],

$$\frac{dP_B}{dt} = (1 - P_B^2) f dP_e \hat{\sigma}. \quad (14)$$

Here f is the orbit frequency of the beam (of order 10^6 Hz), d the target thickness (particles per unit area), P_e the electron polarization and $\hat{\sigma}$ the spin dependent cross section averaged over the orientation of the scattering plane. Note, that Eq. (14) implicitly contains the equality of Eq. (5). For an electron target polarized transverse to the beam we have $\hat{\sigma} = \frac{1}{2}(\sigma_{nn} + \sigma_{mm}) = \sigma_{nn}/2$, while for longitudinal electron polarization $\hat{\sigma} = \sigma_{ll} = \sigma_{nn}$, where σ_{nn} is given by Eq. (13).

Let us now consider polarized target electrons as they occur in a polarized internal hydrogen target. Such a situation was realized recently at the TSR in Heidelberg [6] where a 23 MeV proton beam interacted with an internal target of 6×10^{13} polarized hydrogen atoms per cm^2 . Using Eq. (13), we find for this case $\sigma_{nn} = -140 \text{ mb}$, or $\hat{\sigma} = -70 \text{ mb}$. This implies a large electronic contribution to the beam polarization by a polarized hydrogen target which is of the same magnitude as the experimental result [6], but of opposite sign. This result has to be combined with the effect of the polarized target protons which contribute by selective spin state removal and by Coulomb-nuclear interference scattering, as is discussed elsewhere [9]. Taking this into account as well leads to excellent agreement with the TSR measurement [6,9].

Clearly, the effect of polarized target electrons on the polarization of a stored beam is large enough to be observed. We may thus speculate about its possible use in preparing polarized beams of protons, or, more importantly, antiprotons.

For an atomic hydrogen target there are several ways to increase the polarization rate. For example, one can gain more than a factor of three by polarizing electron and proton in the target opposite to each other, such that electrons and protons contribute with the same sign. Lowering the energy T_{lab} yields a larger electronic spin transfer cross section (see Fig. 1), however, the beam lifetime which is dominated by Rutherford scattering decreases roughly as $1/T_{lab}^2$. Since the electronic spin transfer cross section scales with $1/T_{lab}$, larger energies are preferred, the optimum choice given by machine considerations.

Alternatively, one may consider pure electron targets. One immediate advantage is the absence of nuclei in the target which greatly increases the beam lifetime, bounded only by the quality of the ring vacuum. This should be very important for polarizing antiprotons since

it avoids the difficulty with the large annihilation cross section. Also, the larger screening length in an electron gas raises the spin cross section by about a factor of three (see Fig. 1).

A pure electron target may be realized either by a comoving electron beam, similar to an electron cooling arrangement, or by the trapping of electrons in a potential well.

Although a comoving polarizing electron beam may be similar to an electron cooling beam, the design constraints are different, and a low beam temperature may be traded for intensity. Clearly, such an arrangement requires a high-intensity, high-duty-factor, polarized electron beam, but the rapid developments in this field of research [10] indicate that this technical limitation may be overcome. One clear advantage of a comoving beam is the possibility to choose the best relative energy between the protons and the electrons, independent of the energy of the stored beam. This energy is about 5 MeV (Fig. 1).

It may be possible to produce dense electron targets by plasma confinement techniques [11]. Trapping of electrons in a Penning trap has been used as a diagnostic tool in the Indiana Cooler. A realistic extrapolation of the performance of such a device yields a target thickness of 10^{12} electrons per cm^2 . This is still less than the electron thickness of an internal hydrogen target. But the task of developing a high-density electron trap for use in nuclear physics is new and one has to look to future advances in this area, perhaps borrowing from techniques developed for plasma confinement in nuclear fusion. For a trap in which electrons can be *accumulated*, the requirement on the source of polarized electrons is less demanding. Since the electrons have low velocity in the lab, the choice of beam energy is critical, trading off the spin transfer cross section against the lifetime of the beam.

There continues to be great interest in polarized antiproton beams for nuclear and particle physics research. Unfortunately, none of the many methods to polarize antiprotons which have been suggested [12] has proven practical. In $\bar{e}p$ scattering the spin transfer cross section for \bar{p} is almost as large as for p , (Fig. 1). Therefore polarized electron targets may be useful in polarizing antiprotons.

In this paper we have found a surprisingly large spin-dependent electromagnetic effect. It arises from an interference of the hyperfine amplitude, containing the product of the magnetic moments of the electron and the proton with the Coulomb amplitude. The effect is enhanced because of the singularity at $q=0$ and because of the large m_p/m_e mass ratio. The present calculation is supported implicitly by the result of the TSR experiment [6,9]. It would be desirable to test it further by a similar experiment where the contribution from the electrons is *singled out* by either using a polarized hydrogen target in atomic spin states that differ only in the polarization of the electron, by using a mixture of spin states with polarized electrons, but zero proton polarization, or by choosing the beam energy such that the polarizing effect from the pp interaction is small. In any case it should be possible to carry out such a test using currently existing equipment.

In the present work we use a simple estimate for the effect of screening. Clearly, at small scattering angles and low energies this part of the calculation is critical for atomic hydrogen as well as for pure electron plasma targets. It is important that the treatment of screening is refined and that the role of distortions and of external magnetic fields is studied in more detail.

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FIGURES

FIG. 1. Spin transfer cross section σ_{nn} in $p\vec{e}$ scattering versus beam kinetic energy T_{lab} . The dotted curves are plane wave Born approximation (PWBA) results, Eq. (11), which are the same for protons and antiprotons, while the solid and dashed curves are approximate distorted wave Born approximation (DWBA) calculations, Eq. (13), for protons and antiprotons, respectively. The upper three curves give the electronic contribution for an atomic hydrogen target while the lower curves are for a pure electron target.

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